Axial Mixing and Extraction Efficiency

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The effect of back mixing of either phase in an extraction column, which decreases the extraction efficiency, is analyzed theoretically by means of an idealized diffusion model that can be characterized by four dimensionless parameters: a Peclet number of each phase, a mass transfer number, and the usual extraction factor. Calculations for a wide range of these parameters were performed on a digital computer.

The principal results, presented in a table, will be useful in the design and scale up of extraction columns and in the interpretation of experimental results from extractors and from some reactors in which a first-order reaction occurs.

Circulation and back mixing, or axial mixing, in extraction columns reduce the number of theoretical stages (efficiency). If the back mixing is very severe, the column may extract less than would one theoretical stage. Ideally each phase should flow with no axial mixing; that is it should be in plug flow. In practice, however, there is always some axial mixing. For most extraction-column designs, operating conditions that give high rates of interphase mass transfer give appreciable axial mixing. Failure to account for axial mixing may lead to column-height or packing-efficiency correlations that scatter badly or that may give large errors when extrapolated to large packing or column diameter. In this paper the effect of axial mixing on the efficiency of countercurrent extraction columns is analyzed theoretically by means of an idealized diffusion model.

Despite the realization that back mixing decreases extraction efficiency, there was little mention of it in the literature until 1950. In that year Morello and Poffenberger (*7*) and Geankoplis and Hixson (*2*) noted the existence of back mixing, especially in spray towers. The significance of the mixing to extraction efficiency, however, was not clearly stated until 1952 (*8*). Later work (*1, 3, 4*) included experimental measurements of local mass transfer coefficients and concentration gradients in extraction columns in which there was appreciable back mixing of the continuous phase. All this work was carried out in spray or packed columns. In 1954 Vermijs and Kramers (*12*) noted some effects of back mixing in a rotating-disk column. In no case, however, was there any attempt to characterize quantitatively the axial mixing of either phase. Furthermore at the time this work was begun there had not appeared an experimental or theoretical analysis relating efficiency to back mixing.

A recent Atomic Energy Commission report (*6*) gives some theoretical results for special cases, and a supplement to the report (*5*), which appeared as this paper was being prepared, contains many numerical results. The equations given in these reports are essentially the same as those given here. Furthermore axial concentration profiles are tabulated in the supplement (*5*); whereas in this paper only the exit concentrations are tabulated. It was nevertheless thought worthwhile to publish the results given here because a somewhat broader range of parameters was investigated, the interpretation of the assumptions and applicability of the results is different, and the method of solving the equations is quite different. In the A.E.C. report the emphasis is on the use of the analysis for interpretation of experimental results, whereas in this paper emphasis is on the scale-up problem.

ANALYSIS

The effect of back mixing on efficiency is presented here for continuous, countercurrent extraction with feed and solvent introduced at opposite ends of the column. The calculation is based on a three-component system and an idealized diffusion model, the eight principal assumptions in the analysis being

1. The back mixing of each phase may be characterized by a constant axial diffusion coefficient.
2. The end conditions usually assumed (*13*) for models of this type are applicable [Equations (4) through (7)].
3. The mean velocity and concentration of each phase are constant across that part of the column diameter which is occupied by the phase.
4. The solvent and solute-free raffinate are immiscible (or their solubility does not vary with solute concentration and hence height).
5. The volume rates of the solvent and feed phases do not change with height.
6. The distribution coefficient (equilibrium ratio) is constant; that is, it is not a function of concentration.
7. The product of the mass transfer coefficient and the interfacial area per unit tower volume is constant throughout the column.
8. The gradients of solute concentration in each phase are continuous; that is, there are no discontinuities as would occur in a series of discrete well-mixed stages.

Figure 1 shows the notation and the model based on these assumptions. More detailed notation and the notation of Miyauchi (*6*) are given at the end of the paper. Material balances around each

phase in a differential slice of the column are

$$e_f \frac{d^2x}{dz^2} - V_f \frac{dx}{dz} = Ka(x - my) \quad (1)$$

$$e_s \frac{d^2y}{dz^2} + V_s \frac{dy}{dz} = -Ka(x - my) \quad (2)$$

The appropriate boundary conditions (assumption 2), which are determined by material balances around each phase at each end of the column and have been discussed at some length in the literature (*13*), are

$$-e_f \frac{dx}{dz} + V_f x = V_f x_i \quad (3)$$

$$\frac{dy}{dz} = 0 \quad (4)$$

at $z = 0$

$$e_s \frac{dy}{dz} + V_s y = V_s y_i \quad (5)$$

$$\frac{dx}{dz} = 0 \quad (6)$$

at $z = L$

These equations are more convenient if put in dimensionless form by use of the groups F, P, R, and T. (See Notation: the departure from A.I.Ch.E. Standard Nomenclature for the Peclet numbers, P and R, and the number of transfer units, T, is desirable because of the length of the equations that follow.) The solution is designated by $\psi(w)$, a dimensionless concentration.

At the feed inlet where $x = x_i$, $\psi(w) = \psi(0) = 0$. Furthermore $\psi(1) = 1$ corresponds to $x = my_i$, denoting a raffinate phase that is in equilibrium with solvent of the given concentration. Thus in any column ψ may vary from 0 to a possible maximum of 1, and it denotes the amount or extent of extraction. $\psi(1)$ is the usual recovery fraction as used in absorption-factor calculation methods.

The concentration of the solvent phase is made dimensionless by a similar transformation and is designated by $\Gamma(w)$.

Substitution of the definitions into Equations (1) through (6) yields

$$\psi'' - P\psi' = TP(\psi - 1 + F\Gamma) \quad (7)$$

$$\Gamma'' + R\Gamma' = TR(\psi - 1 + F\Gamma) \quad (8)$$

$$-\psi' + P\psi = 0 \quad (9)$$

$$\Gamma' = 0 \quad (10)$$

at $w = 0$

$$\Gamma' + R\Gamma = 0 \quad (11)$$

$$\psi' = 0 \quad (12)$$

at $w = 1$

The solution of these equations is

$$\psi(w) = A + Be^{\lambda_1 w} + Ce^{\lambda_2 w} + De^{\lambda_3 w} \quad (13)$$

where λ_n are the roots of

$$\lambda^3 + (R - P)\lambda^2 - (TP + PR + TRF)\lambda + TPR(F - 1) = 0 \quad (14)$$

and A, B, C, and D are determined by the following set of four linear algebraic equations:

$$AP + B(P - \lambda_1) + C(P - \lambda_2) + D(P - \lambda_3) = 0 \quad (15)$$

$$B(\lambda_1^3 - P\lambda_1^2 - TP\lambda_1) + C(\lambda_2^3 - P\lambda_2^2 - TP\lambda_2) + D(\lambda_3^3 - P\lambda_3^2 - TP\lambda_3) = 0 \quad (16)$$

$$B\lambda_1 e^{\lambda_1} + C\lambda_2 e^{\lambda_2} + D\lambda_3 e^{\lambda_3} = 0 \quad (17)$$

$$ATPR + Be\lambda^1[TPR + (TP - PR)\lambda_1 + (P - R)\lambda_1^2 - \lambda_1^3] + Ce\lambda^2[TPR + (TP - PR)\lambda_2 + (P - R)\lambda_2^2 - \lambda_2^3] + De\lambda^2[TPR + (TP - PR)\lambda_3 + (P - R)\lambda_3^2 - \lambda_3^3] = TPR \quad (18)$$

For the particular case $F = 1$ Equations (6) through (11) are indeterminate, and the solution is given by a slightly different set of equations, which however will not be given here.

For design purposes it is usually sufficient to know the concentration of the raffinate and of the fat solvent rather than complete concentration profiles; therefore only $\psi(1)$ is given in this paper. $\Gamma(0)$ can be found by a material balance around the column, which is simply

$$\Gamma(0) = \psi(1) \quad (19)$$

For the slug-flow case ($e_f = e_s = 0$, $P = R = \infty$) the extraction is given by

$$\psi_\infty(1) = \frac{1 - e^{T(F-1)}}{1 - Fe^{T(F-1)}} \quad (20)$$

For a single, well-mixed stage ($e_f = e_s = \infty$, $P = R = 0$) the extraction is given by

$$\psi_0 = \frac{T}{1 + T + FT} \quad (21)$$

MACHINE CALCULATION

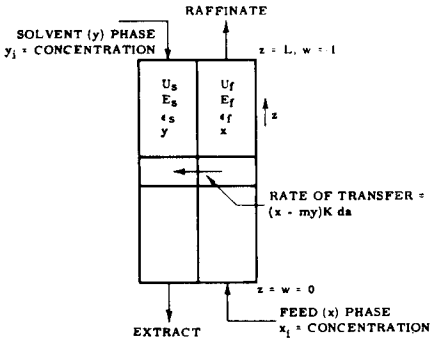
The differential equations (7) and (8) with their boundary conditions (9) through (12) can be solved on a digital computer in two basic ways. First, the differential equations can be approximated by difference equations; second, the coefficients and exponents of the solution equation (13) can be evaluated. The second method was used because it is more accurate for this problem and easier to code.

Figure 2 is a block diagram of the program used to calculate the $\psi(1)$. Floating-point operation was used and eight significant figures were carried.

The previous section mentions that Equations (7) through (12) are indeterminate for $F = 1$. To avoid writing a separate program for this case, 1.001 was used instead of 1 for F.

TABULATION OF RESULTS

Table 1 contains the principal results of the present analysis in the form of $\psi(1)$ as a function of F, P, R, and T for P and R of 5, 10, and 55, which is the range of greatest interest in extraction.



U_f, U_s = ACTUAL MEAN VELOCITY OF FEED AND SOLVENT PHASE
E_f, E_s = EDDY DIFFUSIVITY OF FEED AND SOLVENT PHASE
ϵ_f, ϵ_s = VOLUME FRACTION OF FEED AND SOLVENT PHASE
x, y = CONCENTRATION OF FEED AND SOLVENT PHASE

Fig. 1. Nomenclature for back-mixing model.

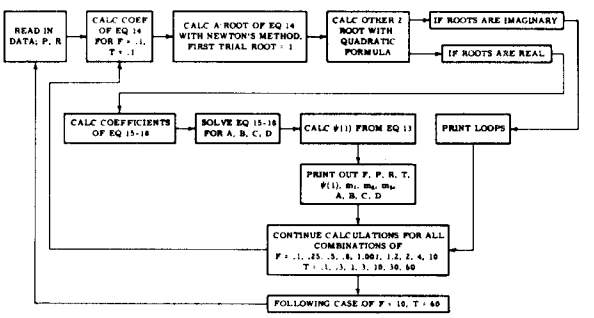


Fig. 2. Block diagram of calculation program.

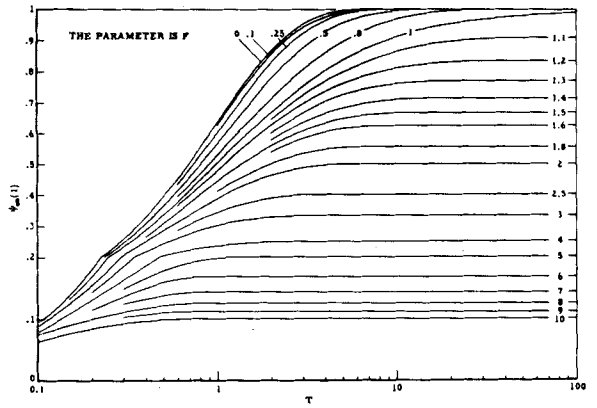


Fig. 3. Extraction with plug flow of each phase (no back mixing, $P_{\infty} = R = \infty$).

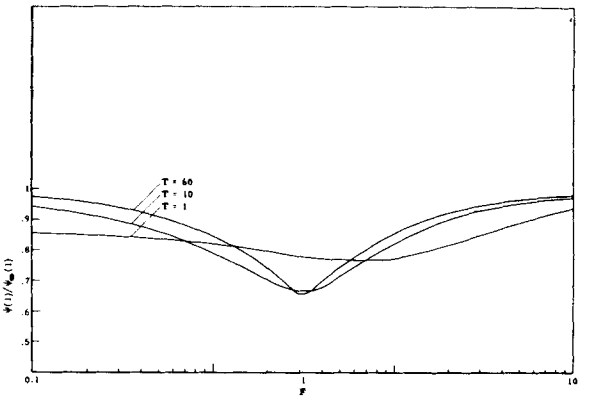


Fig. 4. Change of $\psi(1)/\psi_{\infty}(1)$ with F, $P = R = 2$.

A more complete table has been deposited with the A.D.I.* In the A.D.I. table the two rather special cases for $F = 0$ and for $P = 55$ with $R = .001$ are at the head of the table, whereas the remaining cases are tabulated in an orderly pattern. The solutions for $P = R = \infty$ and $P = R = 0$ have not been tabulated, but they are easily found from Equations (20) and (21). For reference the former is graphed in Figure 3, and the latter is within 1% of the case of $P = R = .05$.

The range of P and R covered is .05 to 55. The exact solutions for one well-mixed stage ($P = R = 0$) differ from the solutions for $P = R = .05$ by less than 1%. At high P however the $\psi(1)$ for $R = .05$ and $R = 0$ differ by more than 1%, and so in this range $\psi(1)$ was calculated for $R = .001$ and .0001. These solutions differ by less than 0.1% and therefore probably differ from the solutions for $R = 0$ by much less than 1%. Solutions for $R = .001$ are given in the A.D.I. table. At high P however it is unlikely that R could be as low as .05. Thus for practical purposes Peclet numbers of .05 and 0 are equivalent. Values of P and R that are higher than 55 are not reported, because they could not be calculated without writing another program. Now $\psi(1)$ varies most strongly with high R when $F = 1$ and P is large. In this region $\psi(1)$ for $P = R = 55$ differs from $\psi_{\infty}(1)$ by at most 5%. For F less than .8 or greater than 1.2 the difference is less than 2.4%. Thus solutions for P and R of 55 are quite close to the limiting case of P and R equal to infinity.

The calculations cover a range of T between 0.1 and 60. It is rare if ever that T is less than 0.1. It may occasionally be greater than 60, but the difference in $\psi(1)$ between $T = 60$ and $T = \infty$ is very small. At low P and R the difference is a maximum at $F = 0$, where it is 1.6%; at high P and R the difference is a maximum at $F = 1$, where it is also 1.6%.

The range of F covered is 0 to 10. Although the case of $F = 0$ is of little interest in extraction, it is included because it corresponds to a reactor in which rate-controlled, first-order reactions of constituents of one phase are catalyzed by a second immiscible phase. The solutions for $F = 0.8$ and $F = 1.2$ were found because of the rather rapid change of $\psi(1)/\psi_{\infty}(1)$ with F near $F = 1$, which is illustrated in Figure 4 for $P = R = 2$ and three values of T. Because of this behavior it was thought that plots of $\psi(1)$ vs. F might show peculiarities in the vicinity of $F = 1$, but such is not the case.

The density of parameters within the range discussed above is sufficient to allow interpolation with a precision of 1 or 2%. For interpolation in F near $F = 1$ it is suggested that plots of $\psi(1)$ vs. F be used; for interpolation in F with F not near unity and for interpolation in T, P, and R usually greater precision is obtained from plots of $\psi(1)/\psi_{\infty}(1)$ vs. the parameter. Such plots have the added conceptual advantage that $\psi(1)/\psi_{\infty}(1)$ is a direct measure of the extent to which back mixing decreases extraction; the ratio may be considered a back mixing efficiency.

Accurate interpolation among the four parameters is often required because extractor height is sensitive to the extraction required. Such interpolation is of course laborious and time consuming. A correlation of the results has therefore been developed in which interpolation in F only is required. The correlation uses the ratio T_p/T, where T_p is that value of T which will accomplish the given extraction in a column with no axial mixing. Now the ratio T_p/T is equal to L_p/L, the ratio of height of a plug-flow column to the actual height, and it will therefore be called *column efficiency* E_c. A column efficiency of 0.7 can be interpreted to mean that $1 - 0.7 = 0.3$ is the fraction of required height that is necessary to overcome the effect of axial mixing. If E_c is known, T_p can be calculated and used in Equation (20) to find the extent of extraction ψ.

The correlating equation for E_c is

$$E_c = \frac{PR}{PR + T[aP + bR + c\sqrt{PR} - d\sqrt{P + R} + g(P - R)e^{-hT}]} \tag{22}$$

where the constants are the following functions of F only:

F	a	b	c	d	g	h
0.1	0.43	0.15	0.31	0.41	−0.305	0.073
0.2	0.47	0.21	0 485	0.59	−0.29	0.085
0.3	0.495	0.27	0.625	0.73	−0.255	0.094
0.4	0.515	0.32	0.75	0.85	−0.22	0.100
0.6	0.55	0.425	0.975	1.07	−0.15	0.115
0.8	0.58	0.52	1.16	1.25	−0.075	0.125
1.0	0.61	0.61	1.31	1.42	0	0.135
1.5	0.675	0.81	1.61	1.78	0.18	0.155
2.0	0.73	1.00	1.85	2.10	0.35	0.172
3.0	0.80	1.38	2.12	2.45	0.64	0.201
4.0	0.845	2.00	2.25	2.65	0.865	0.225

This empirical equation gives a maximum error of 10% in E_c and 1.5% in ψ in the range $0.1 \leq F \leq 4$, $2 \leq P$ and $R \leq 55$, $1 \leq T \leq 60$. These errors are well within the accuracy justified by the assumptions in the analysis. The equation should not be used outside the tested range of parameters, and in particular it should not be used for Péclet numbers less than about 1 or 2.

Figures 5 and 6 illustrate the behavior of extraction efficiency for certain sets

*Tabular material has been deposited as document 5875 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $1.25 for photoprints or $1.25 for 35-mm. microfilm.

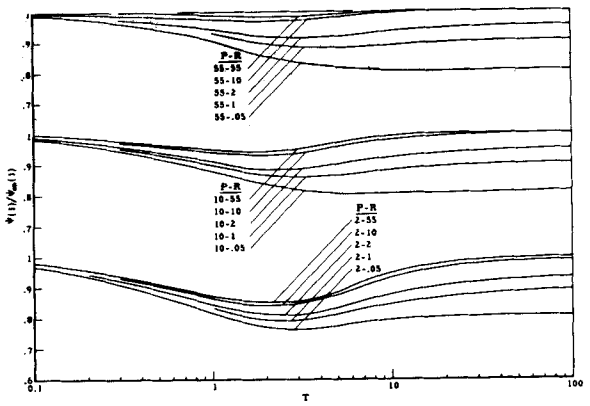


Fig. 5. Effect of back mixing on efficiency, $F = 0.25$.

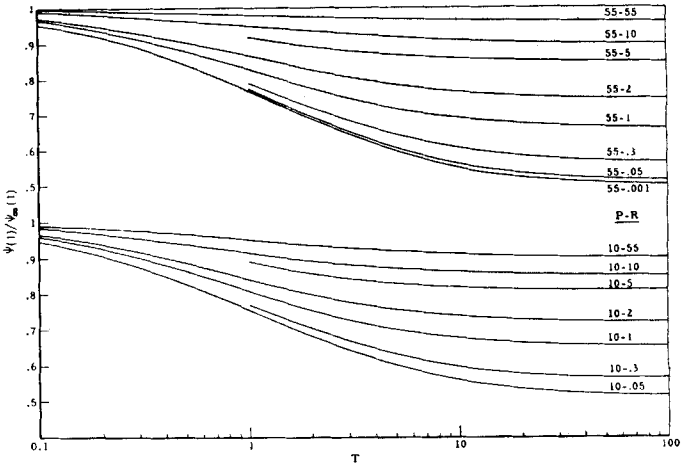


Fig. 6. Effect of back mixing on efficiency, $F = 1$.

of parameters. For F different from unity the effect of back mixing on efficiency goes through a minimum as T is increased (Figure 5). In no region of parameter space, however, does back mixing reduce the extent of extraction by more than 50%. Even for the rather low Peclet numbers of $P = R = 5$, the extraction is reduced by less than 25%. On the other hand back mixing may have a large effect on the height necessary to accomplish a given extraction, especially when the extent of extraction is large. Suppose for example one desires to extract to the extent of $\psi(1) = 0.98$ with $F = 0.25$. With no back mixing in either phase, this can be done with $T = 4.8$. Suppose however that for the same height of tower that gives $T = 4.8$, one finds that $P = R = 10$, a combination which gives $\psi(1)/\psi_\infty(1) = 0.952$ or $\psi(1) = (0.952)(0.98) = 0.933$, a 5% reduction due to back mixing. To increase this to 0.98 it is necessary to increase L by about 60%, so that $P = R = 16$ and $T = KaL/V_f = 4.8\ (16/10) = 7.7$.

USE OF ANALYSIS

The analysis will be useful both in the interpretation of extraction data and in the scale up of extractors. The use of this model in the interpretation of experimental data has already been noted by Miyauchi (*6*), who has emphasized that three different definitions of T are possible. Their magnitudes differ widely; yet all have been indiscriminately referred to, even in recent literature, as the "number of transfer units, NTU." In most cases the authors have not mentioned the possibility of two or three conflicting definitions. The definition used in this report is called the *true* value by Miyauchi. It is defined by

$$T = \frac{KaL}{V_f}$$

The two apparent values of NTU are called the *measured* value

$$T_M = \int_{x(0)}^{x(L)} \frac{dx}{x - my}$$

and the *piston-flow*, or *plug-flow*, value, which is defined by Equation (20). The apparent values of T defined above include effects of axial mixing, and in general

$$T \geq T_M \geq T_P$$

To illustrate the difference between the values of these numbers, Miyauchi calculated them for the particular case $F = 1$, $P = R = 4$, and $T = 5$. The results are $T_M = 3.86$ and $T_P = 1.76$. It is apparent that failure to include axial mixing as a variable in correlations for T and L/T (height of a transfer unit $= HTU$) is a major cause of difficulty, scatter, and discrepancies among various investigations.

This report emphasizes less the interpretation of experimental data than the application of the analysis to problems of scale up. For this purpose knowledge of the concentration distribution in an extractor is not usually necessary, for it is only the composition of the exit streams that is of interest. The present analysis is a step in a logical sequence in the scale up of extractor height. It presupposes a knowledge of the effect of a change in column diameter and internals on Ka, E_f, and E_s. With this knowledge the results provide a means by which these variables may be combined to compute the height necessary to accomplish a given extraction with given

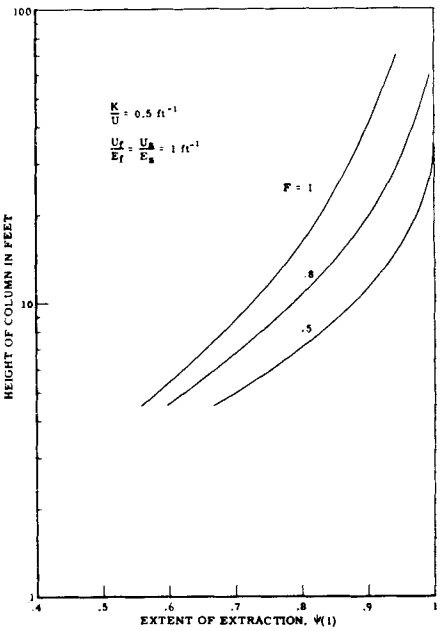


Fig. 7. Effect of column height on extent of extraction.

TABLE 1. $\psi(1)$ AS A FUNCTION OF F, P, R, AND T.

P = 5 R = 5

F	T = 1	3	10	30	60
.1	.565	.860	.981	.9953	.9970
.25	.559	.815	.950	.979	.984
.5	.499	.740	.879	.923	.935
1	.431	.609	.708	.747	.758
2	.351	.418	.496	.467	.479
4	.216	.2396	.2494	.2468	.2471
10	.0981	.0999	.0998	.0998	.0998

P = 10 R = 5

F	T = 1	3	10	30	60
.1	.584	.886	.989	.9970	.9979
.25	.557	.843	.966	.986	.9900
.5	.516	.770	.906	.944	.994
1	.444	.653	.741	.788	.794
2	.340	.452	.472	.485	.486
4	.220	.244	.2487	.2494	.2495
10	.0987	.0999	.1000	.1000	.1000

P = 55 R = 5

F	T = 1	3	10	30	60
.1	.606	.917	.9949	.9982	.9986
.25	.578	.877	.981	.9920	.9936
.5	.534	.804	.935	.965	.970
1	.468	.699	.778	.821	.833
2	.368	.446	.486	.4949	.4966
4	.223	.2464	.2498	.2500	.2500
10	.0991	.1000	.1000	.1000	.1000

P = 5 R = 10

F	T = 1	3	10	30	60
.1	.568	.866	.987	.9991	.9997
.25	.545	.829	.967	.9928	.9963
.5	.509	.765	.910	.956	.967
1	.444	.631	.741	.782	.794
2	.344	.435	.468	.477	.479
4	.223	.2430	.2470	.2478	.2480
10	.0989	.0997	.0998	.0999	.0999

P = 10 R = 10

F	T = 1	3	10	30	60
.1	.587	.893	.9943	.9997	.9999
.25	.563	.858	.982	.9972	.9986
.5	.526	.795	.940	.977	.984
1	.458	.660	.788	.826	.839
2	.353	.449	.484	.492	.494
4	.227	.247	.2495	.2498	.2498
10	.0994	.1000	.1000	.1000	.1000

P = 55 R = 10

F	T = 1	3	10	30	60
.1	.609	.924	.9987	.9999	1.0000
.25	.584	.893	.9940	.9994	.9997
.5	.545	.832	.970	.9922	.9952
1	.474	.692	.826	.874	.887
2	.363	.463	.4998	.4998	.4997
4	.230	.2487	.2500	.2500	.2500
10	.0996	.1000	.1000	.1000	.1000

P = 5 R = 55

F	T = 1	3	10	30	60
.1	.570	.871	.9906	.9998	1.0000
.25	.551	.841	.979	.9984	.9997
.5	.519	.784	.939	.981	.9897
1	.458	.659	.778	.821	.832
2	.359	.450	.479	.485	.487
4	.230	.2461	.2482	.2486	.2490
10	.0995	.1000	.1000	.1000	.1000

P = 10 R = 55

F	T = 1	3	10	30	60
.1	.589	.898	.9965	1.0000	1.0000
.25	.569	.871	.9909	.9998	1.0000
.5	.536	.818	.967	.9963	.9985
1	.474	.693	.826	.874	.887
2	.370	.467	.4940	.4976	.4982
4	.235	.2491	.2499	.2499	.2500
10	.0999	.1000	.1000	.1000	.1000

P = 55 R = 55

F	T = 1	3	10	30	60
.1	.612	.930	.9996	1.0000	1.0000
.25	.591	.906	.9985	1.0000	1.0000
.5	.557	.858	.9904	.9999	1.0000
1	.491	.729	.879	.933	.948
2	.381	.481	.4997	.5000	.5000
4	.238	.2499	.2500	.2500	.2500
10	.1000	.1000	.1000	.1000	.1000

solvent, feed compositions, and flow rates.

Figure 7 is an example of a kind of plot, derived from the data of Table 1, useful to the design engineer. It shows the height of a column in feet vs. the extent of extraction for three values of F. In this case K/V_F, U_F/E_F, and U_s/E_s have been assumed constant at 0.5, 1, and 1 ft.$^{-1}$ respectively. Such a chart shows clearly the amount that column height must be increased or feed/solvent ratio decreased to effect a given change in extraction.

APPLICABILITY OF THE ANALYSIS

The applicability of the analysis is limited by the assumptions listed earlier. Of these, assumptions 3 through 8 need little comment, except a note that their admissibility must be determined in each case. Assumptions 1 and 2 however merit further discussion.

Assumption 1 is that the extraction process may be represented by a diffusion model. Actually axial mixing in the continuous phase of an extraction column is the result of two effects. The first is true turbulent and molecular diffusion in the axial direction. This type of diffusion can be determined by measuring the concentration upstream from a plane of steady injection of a tracer. Second, axial mixing is caused by nonuniform velocity and subsequent radial mixing, which is sometimes called *Taylor diffusion* after G. I. Taylor's analysis of axial mixing in pipes (*9, 10, 11*). This type of mixing could not be detected by the preceding experiment, but it can be determined, for example, from the spreading of pulse injection of tracers, that is, by measuring the residence-time distribution. Of course this type of experiment will detect the combined effects of the two causes of axial mixing. Both types of diffusion are deleterious in extraction, because both cause a decrease in the net driving force for mass transfer. In the dispersed phase an analogous situation exists. True dispersed-phase diffusion is caused by actual backflow of some drops in a random manner. However the drops also have a spread in residence-time distribution owing to the fact that some drops move faster than others because of the nonuniformity of drop size and of continuous-phase velocity. In addition the residence-time distribution is affected by drop interactions (coalescence, redispersion), which occur even in a nonstaged column.

There are many extraction situations in which Taylor diffusion predominates over eddy diffusion. This would usually occur for example in both phases of packed and spray columns and in both phases of a rotating-disk column operated at high throughput and low rotor speed. In this analysis true eddy diffusion has been assumed, and the question arises as to the conditions under which an eddy-diffusion coefficient can be used to represent the effect of Taylor diffusion in extraction. At present this question cannot be answered; it can only be pointed out that the conditions will depend on other factors, such as the rate of coalescence and redispersion of the dispersed phase and the rate of radial mixing of the continuous phase. Because of the variation of mass transfer rate and concentration with drop size, they will also depend on drop-size distribution. The most reasonable approach at present is to use an eddy-diffusion coefficient that is related to the downstream spread of a tracer, for that is at least a measure of axial mixing if not of its effect on extraction.

The choice of boundary conditions, assumption 2 and Equations (3) through (6), is really a consequence of the use of a diffusion model. This point and the diffusion and boundary equations have been discussed in detail by Wehner and Wilhelm (*13*). An analysis which included Taylor diffusion effects would necessarily employ quite different boundary conditions. As P and R increase, however, the solutions become less sensitive to the choice of boundary conditions. For P and R above about 10 any reasonable boundary conditions will give solutions nearly equal to those for the boundary conditions (3) through (6).

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NOTATION

The letters in parentheses refer to the notation for the equivalent group or parameter of references 5 and 6.

A = $(1 - A_1)$, coefficient in Equation (13)
a = (a), interfacial area between phases per unit-tower volume
B = $(-A_2)$, coefficient in Equation (13)
C = $(-A_3)$, coefficient in Equation (13)
D = $(-A_4)$, coefficient in Equation (13)
E_i = (E_i), eddy diffusivity of phase j, $j = f$ or s
e_i = superficial eddy diffusivity of phase j, $\epsilon_i E_i$
F = (Λ), extraction factor = $V_f m/V_s$
K = (K_x), over-all mass transfer coefficient based on area a
L = column length
m = (m), x/y at equilibrium, distribution coefficient
P = $(P_x B)$, $V_f L/e_f = U_f L/E_f$, Peclet number of feed phase based on column length
R = $(P_y B)$, $V_s L/e_s = U_s L/E_s$, Peclet number of solvent phase based on column length
T = (N_{0i}), KaL/V_f, a mass transfer number sometimes called *the number of transfer units*
T_p = piston-flow, or plug-flow, value
U_j = (u_i), velocity of phase j, $j = f$ or s
V_j = (F_i), superficial velocity of phase j, $\epsilon_j V_j$
w = (Z), z/L, dimensionless distance from feed end of column
x = (c_x), concentration of solute in feed phase
y = (c_y), concentration of solute in solvent phase
z = (z), distance from feed end

Greek Letters

Γ = $(1 - Y)$, dimensionless concentration of solvent phase, $(y - y_i)V_s/(x_i - my_i)V_f$
ϵ_j = (ϵ_i), volume fraction or holdup of phase j, $j = f$ or s
λ_n = (λ_{n+1}), exponent in Equation (13)
ψ = $(1 - X)$, dimensionless concentration of feed phase, $(x_i - x)/(x_i - my_i)$
ψ_∞ = ψ for the case of plug flow $(P = R = \infty)$

Subscripts

f = (x), feed phase
i = (superscript 0 or L), inlet of either phase
s = (y), solvent phase

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